

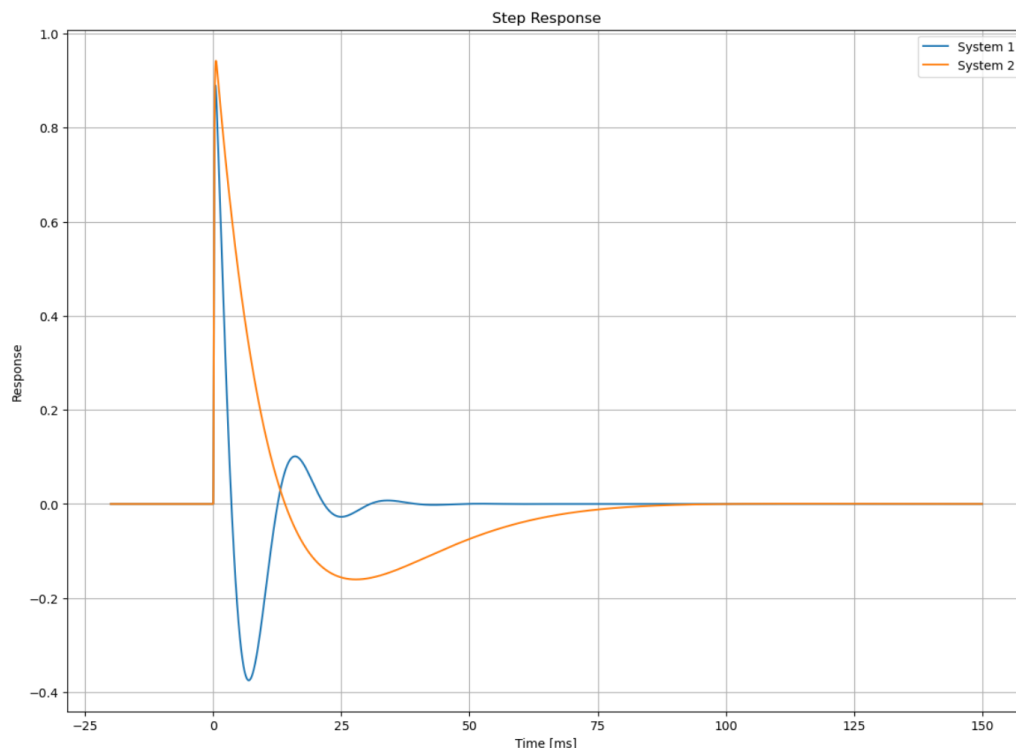
Step Response and Time Domain Behavior for Select Bass Alignments

By: Dan Cyr, January 2024

Introduction:

This paper explores very simplistic models of a handful of bass alignments modelled as various idealistic filter responses. These models are accurate up to a point – and should be considered small signal valid only. The motivation for this paper was repeatedly seeing incorrect interpretation of step response graphs as to time-domain behavior online in many places. In general, sealed boxes are modelled with a second order high-pass filter, while vented, passive radiator and transmission-line enclosures are modelled with fourth-order high-pass filters. For the fourth order systems I used Butterworth filters which have the maximum bass extension without passband ripple

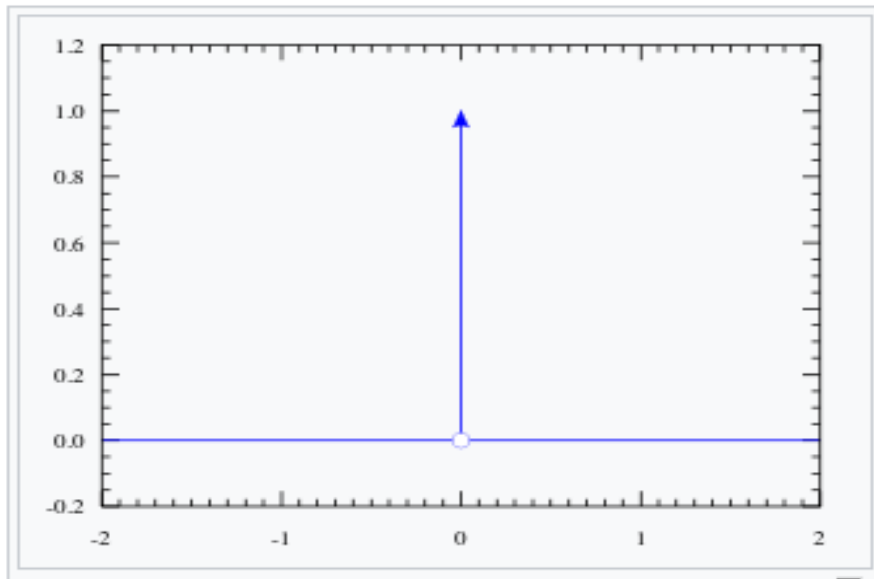
As a teaser, here is the step response of 2 different systems. Which of the two systems would seem to have “faster” bass response to transients? Which would “decay” or “settle” faster? They have identical low-pass filters at 2000hz but are different in the high pass.



Introduction to Impulse and Step Responses of Filters

The impulse response of a filter is defined as its response to an input that is 0 for $t < 0$ and $t > 0$, but equal to infinity at $t = 0$. An infinitely narrow pulse that is infinitely tall is a bit confusing, for the purpose of this paper it is acceptable to envision it as “very narrow and very tall”.

A graph of an ideal impulse sourced from Wikipedia is shown here:



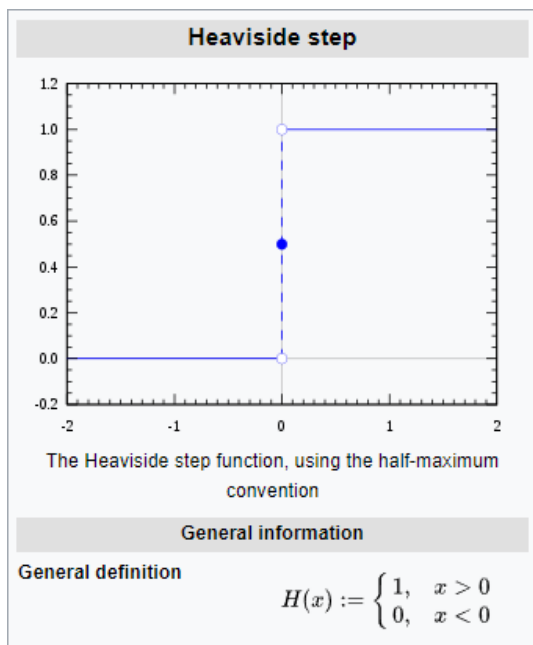
For the calculus savvy, the integral of the ideal impulse (sometimes called the Dirac Delta function) is defined to have area=1. The integral of the impulse response also defines the step response.

Step response is defined as response to input that is 0 for $t < 0$ and 1 for $t > 0$. The value at $t=0$ is treated as any of (0, 0.5, 1), for our case it does not matter. Sometimes called the unit step function or Heaviside step function.

An Aside:

Both the ideal impulse and unit step function are defined mathematically with calculus but also digitally approximated with a DAC. The impulse response in digital realm is all samples take value 0, and at time 0 a single sample of maximum digital value. In practice the impulse response is measured, and the integral performed to calculate the step function, rather than measuring the step response with a step input stimulus.

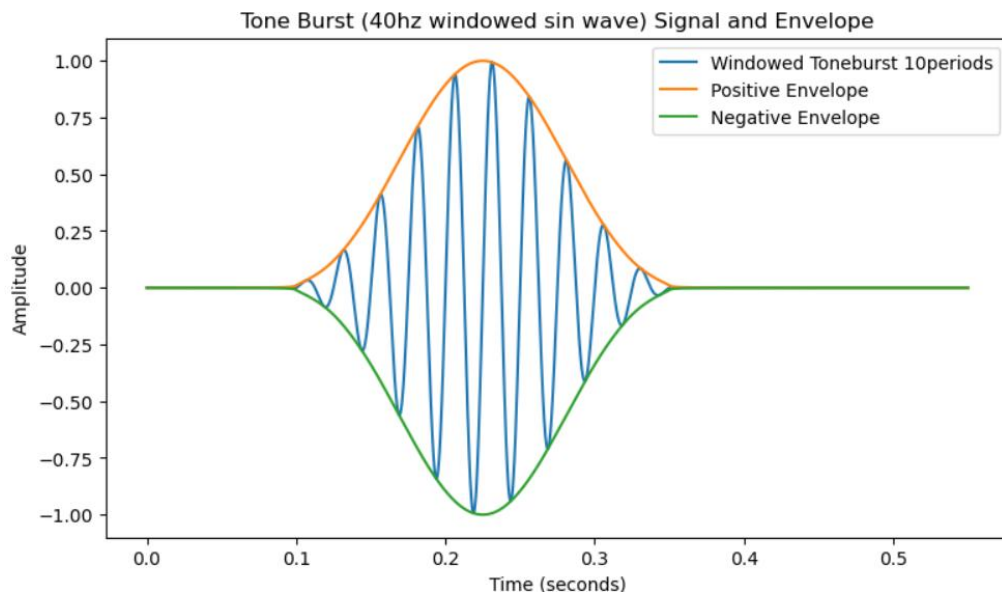
Here is a plot sourced from Wikipedia for the step function:



Some people would understandably pick “System 1” above as “settling” or “decaying” faster but in practice “System 2” has better time-domain performance. Intuitively, a system that lacks bass will have a step-response that settles to zero quicker, after all an infinite bandwidth system’s step response is 1 for all $t > 0$. The time it takes step response to “return to zero” has more to do with its bass extension than time domain performance.

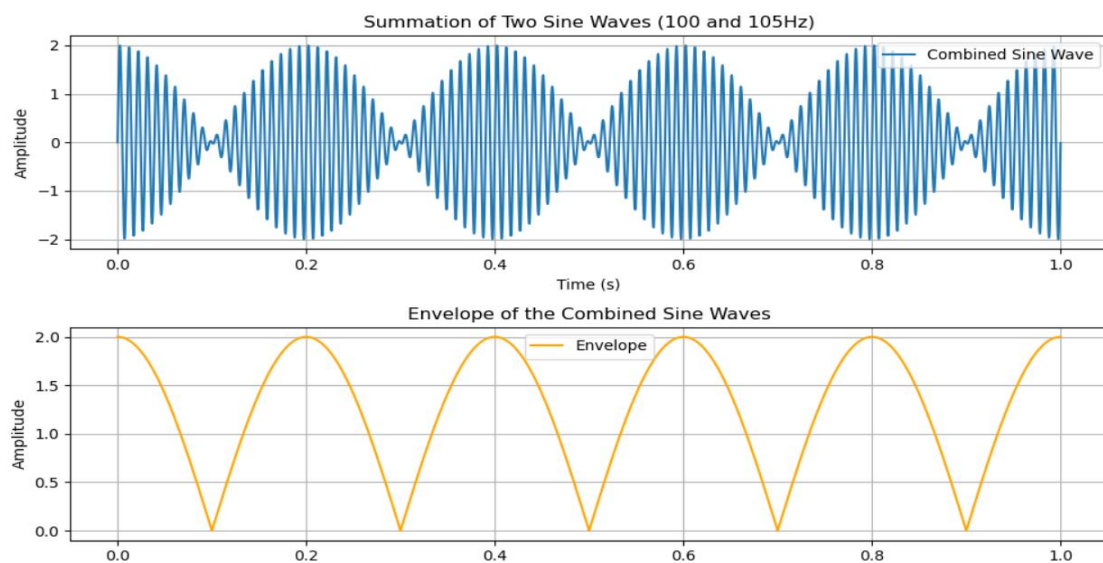
Introduction to the Envelope of Signal (nearly math free!)

I need to introduce the envelope of a signal which is a way of identifying the instantaneous amplitude of a rapidly changing complex signal. A picture is worth more than many words:

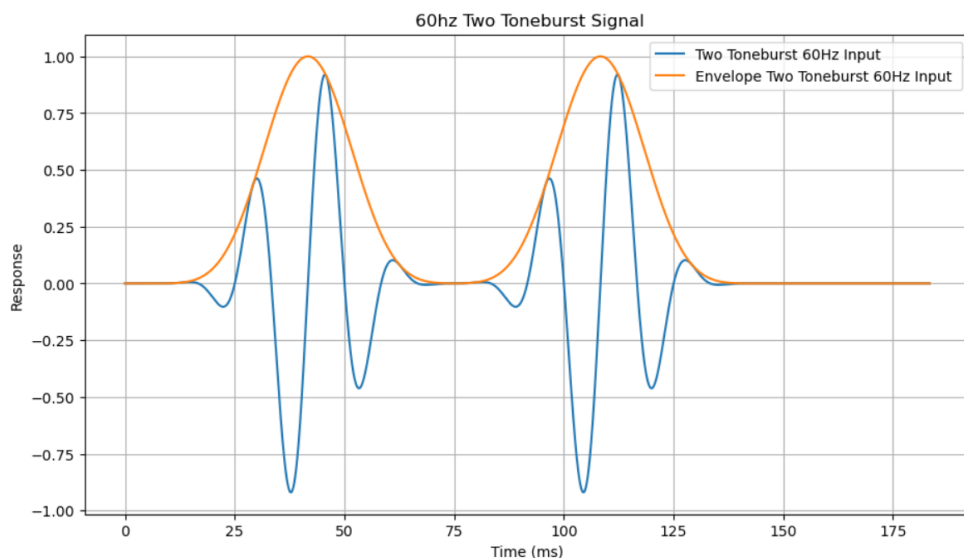


In the above plot, I plotted both the positive and negative envelopes of a windowed (Kaiser) sine wave forming a 10-cycle toneburst. For the rest of the paper, I will only plot the positive envelope, as that is sufficient (so the orange curve above) to see what is happening. The envelope will be useful later when studying the various bass alignments responses to dynamic bass input. If you think this is somewhat like the Energy Time Curve (ETC) frequently seen, you would be correct, both use the Hilbert Transform. The ETC curve is in fact the envelope of the impulse response but plotted differently.

Another example of the envelope of a signal – here is the summation of 2 sine waves (100 and 105Hz) showing the beat-frequency and the envelope shows the beating very clearly.



The toneburst signal (windowed sine wave) is very useful to judging bass time-domain quality, and the envelope is especially useful for judging the amount of “smear” or delay the different filter models have. I am going to use a “two toneburst” signal with 4cycles of each of a given frequency sine wave. Here is the signal and envelope for a 60Hz two toneburst:



The above envelope shows two clearly separated peaks for the envelope, with the envelope falling to 0 in between.

A few selected bass alignments:

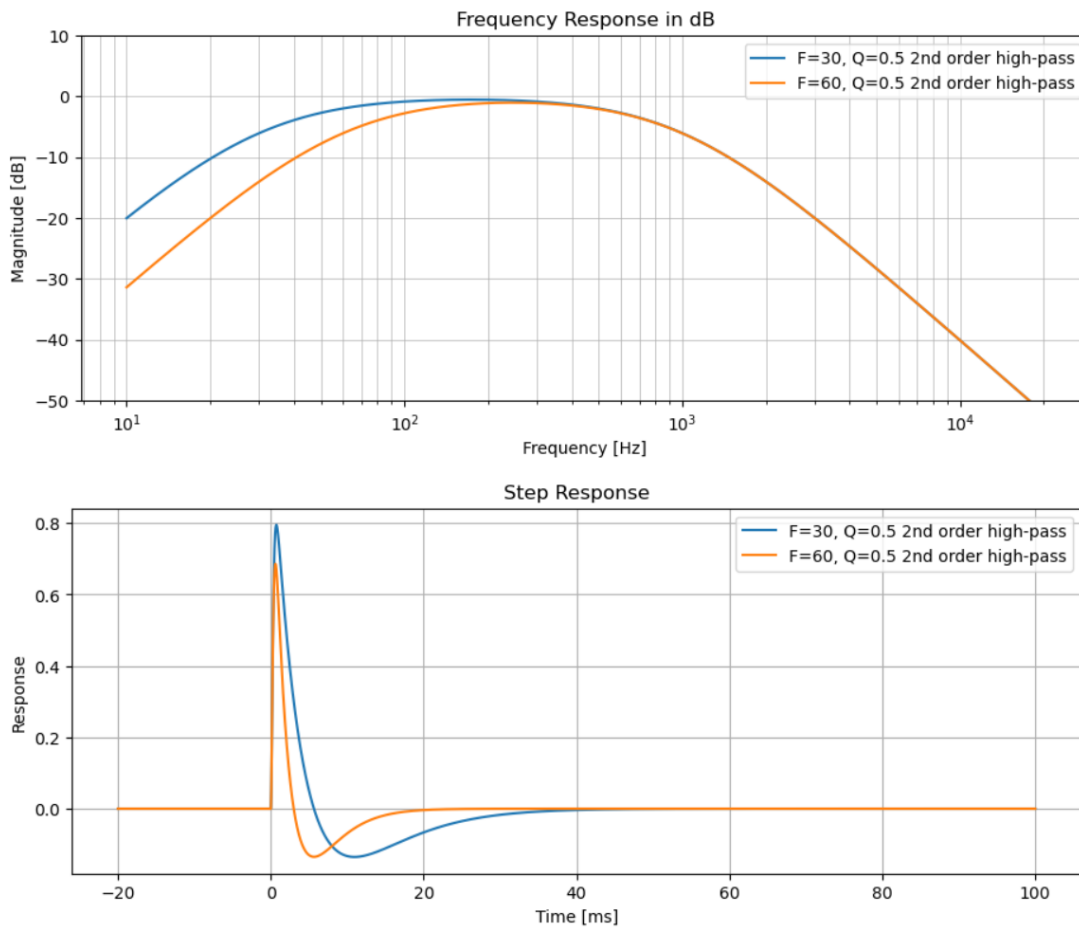
It is now time to look at a few different bass alignments. I have experimented with vented and sealed box alignments for years. One system I built was a sixth-order vented box subwoofer (pre-DSP) which produced deep and flat bass in a small cabinet, but despite measuring well it sounded muddy in comparison to a sealed cabinet with the same driver (also driven with op-amp based Linkwitz transform). In all the following examples a low-pass

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Linkwitz-Riley 2nd order filter is applied at 1000hz so the differences in the responses are from the different high-pass filters. Frequency, step and toneburst responses will be shown, along with the envelope for the tonebursts.

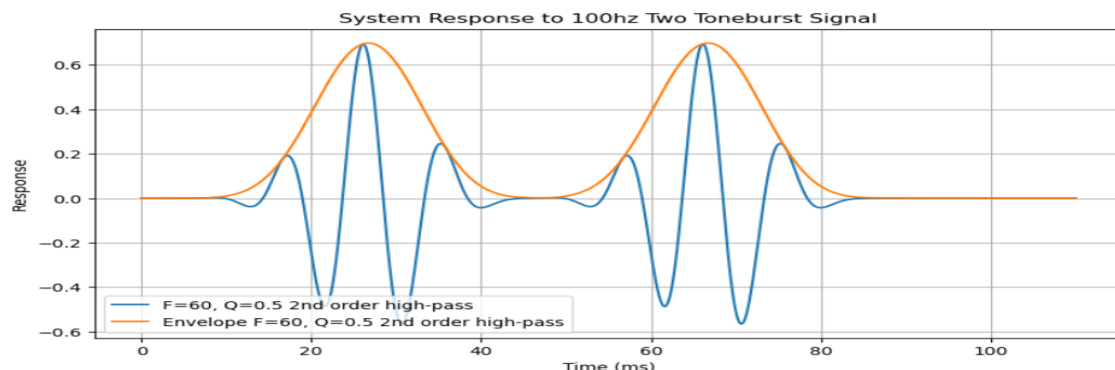
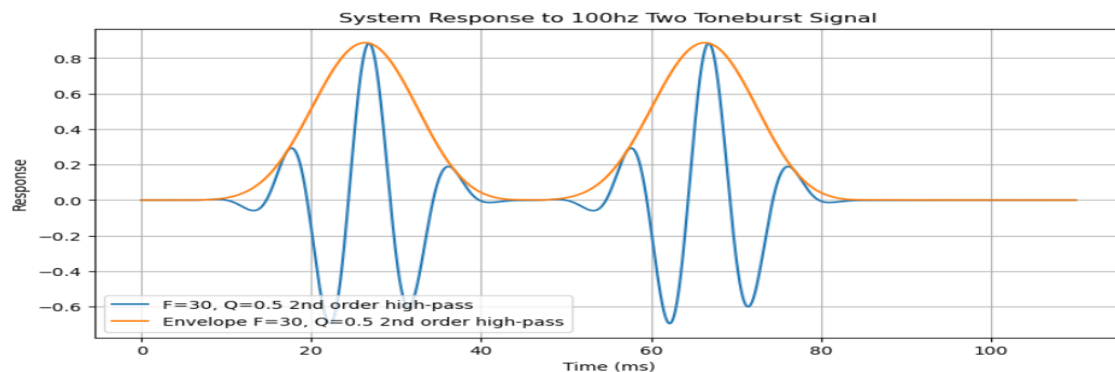
Example 1: Two sealed box – same Q, different Fb

Here are frequency and step response graphs for $Q=0.5$, $Fb1=30\text{Hz}$, $Fb2=60\text{Hz}$:

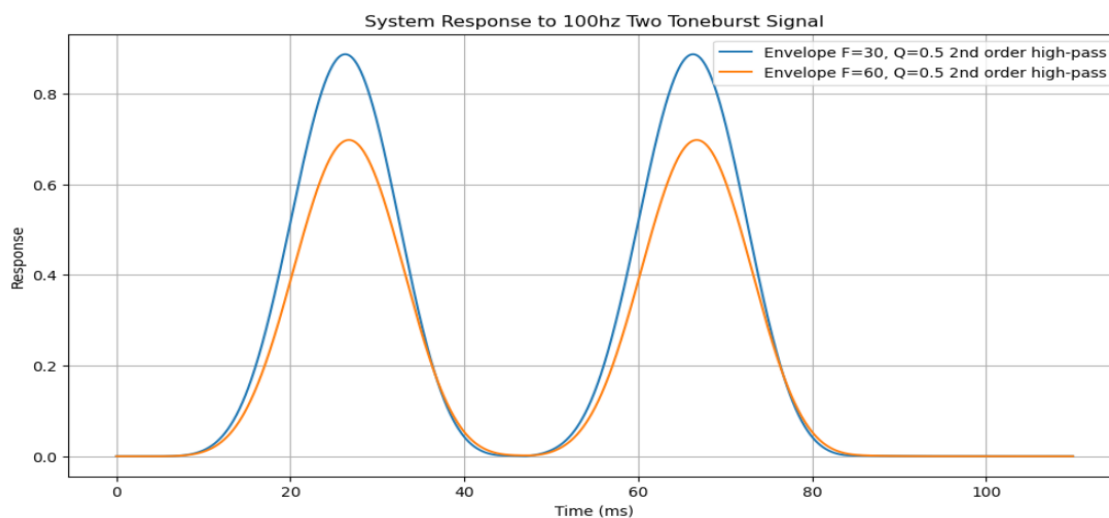


Notice the $F=60$ appears to “settle faster” than the $F=30$ case.

Here are the toneburst responses for 100hz:

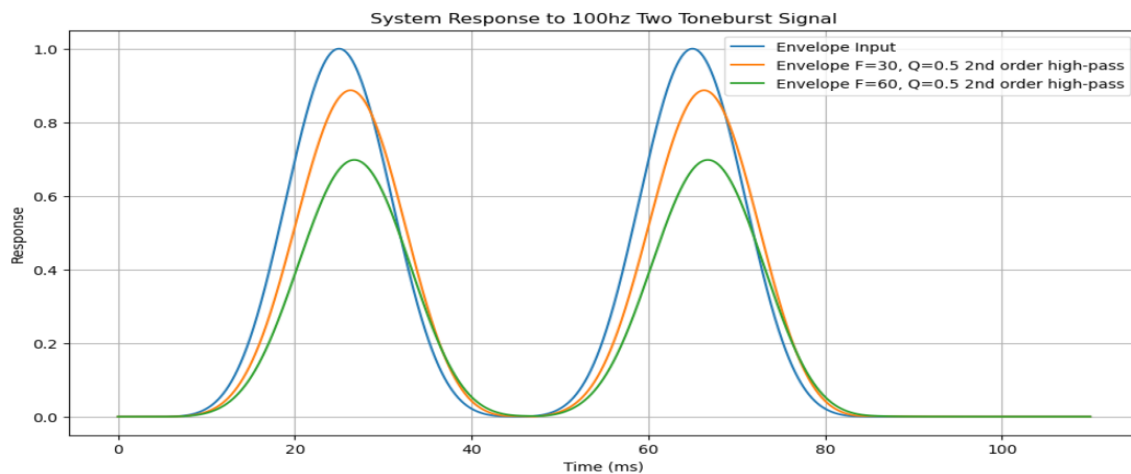


Let's just look at the envelope of each on the same plot:



Remember the step-response for the F=30Hz case took a lot longer to “settle” to 0 and looked like it might decay slower. The envelopes of tonebursts show this isn't the case, the f=60 case has less amplitude because it's closer to the filter knee, and Q=0.5 response has a shallow roll off. The f=60 case is slightly delayed compared to the f=30 case.

How about comparing the output envelope to input envelope?

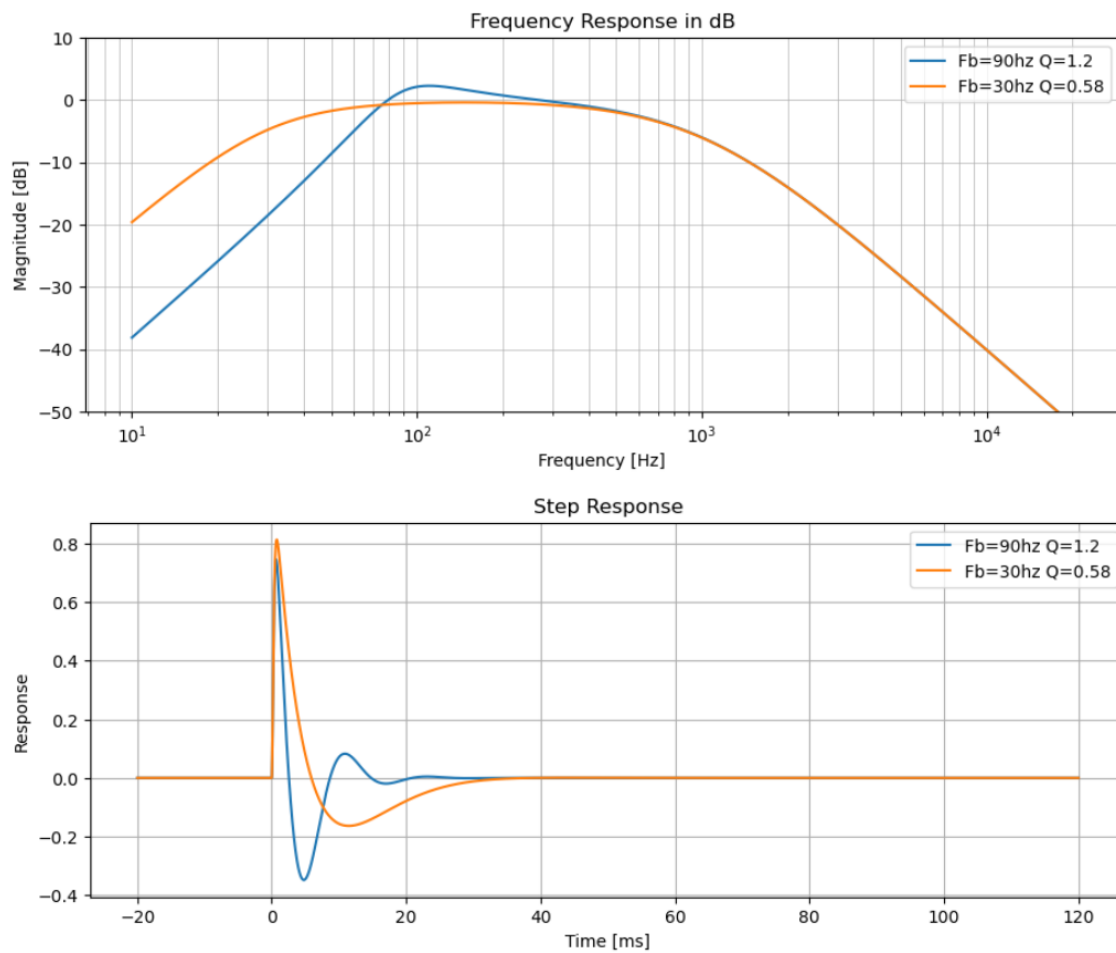


It is clear the output is slightly delayed compared to the input (more so for $F=60$), but the envelope of the output falls to zero and the peaks are just as separated as the input, and decay is excellent.

The important point here – having a system extend lower in frequency does not hurt the transient response, in fact it helps. The reason of course being the group-delay is lower (at the frequency of the toneburst) for the lower corner frequency filter.

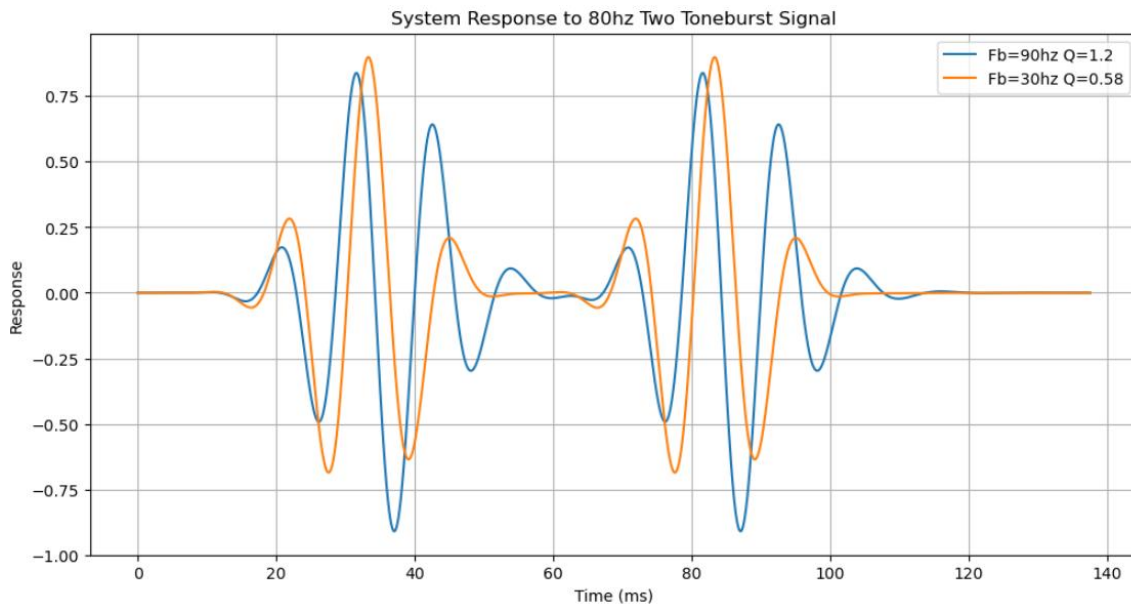
Example 2: Sealed box - Different F_b , Q

Now compare $F=30$ for $Q=0.58$ and $F=90$ $Q=1.2$ – Sometimes designers with small, sealed boxes will use a high- Q to give it some apparent weight in the bass. The LS3/5 is a classic example. First up is frequency and step response:

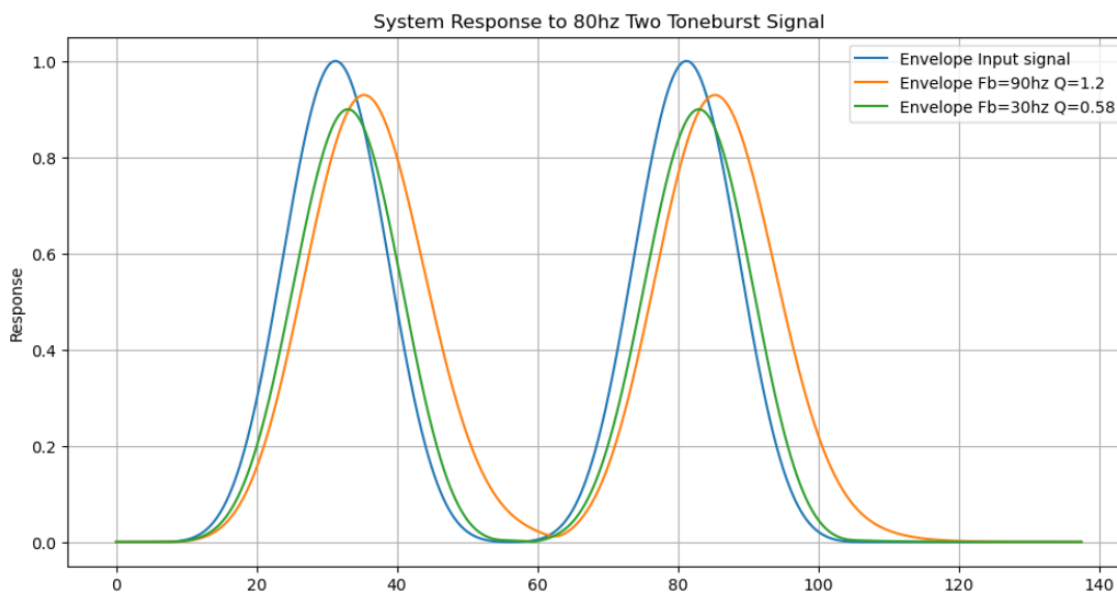


Notice the $Q=1.2$ case the step response decays quicker than the $Q=0.58$ and has a small (<3 dB) peak $\sim 100\text{Hz}$.

Now the toneburst responses:



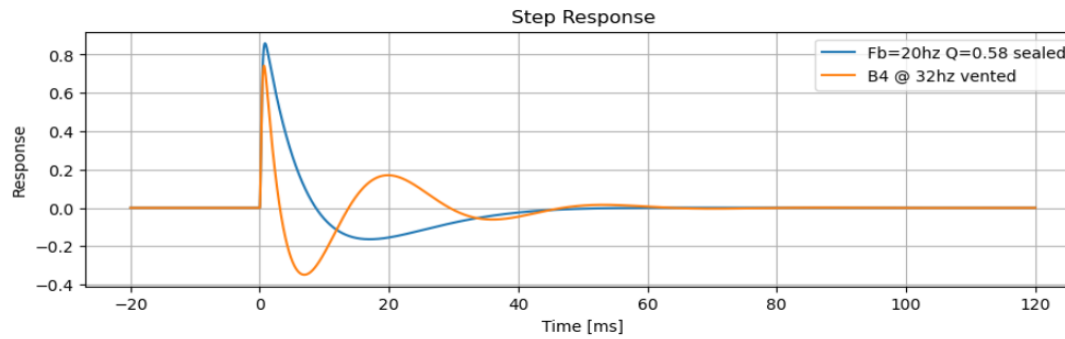
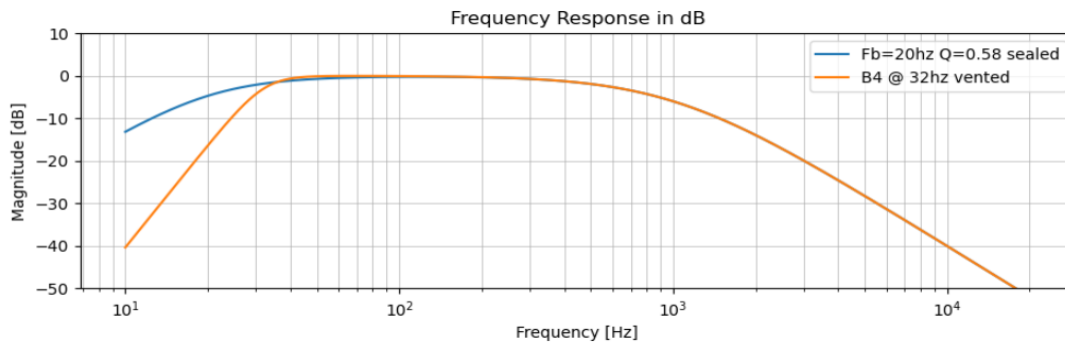
It looks like the $Q=1.2$ case is decaying slower – but envelope shows it clearly:



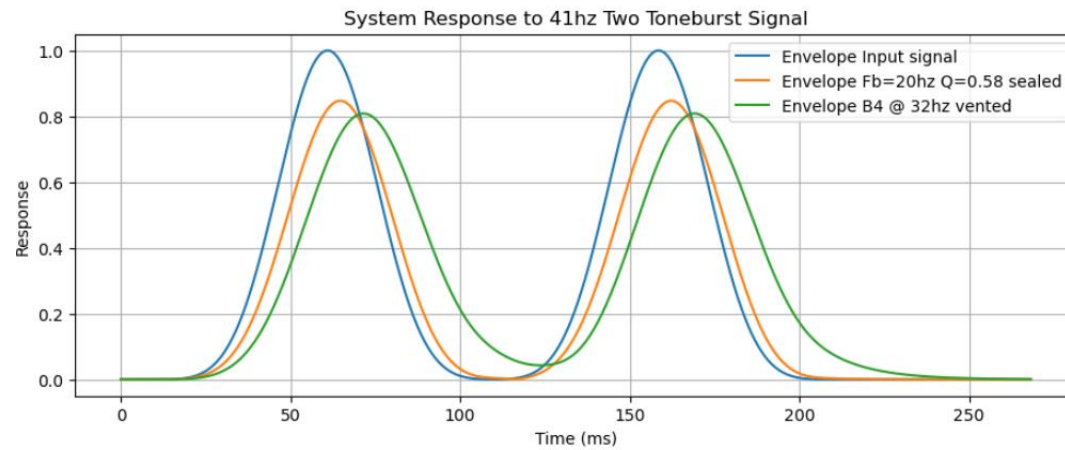
Notice the $Q=1.2$ case has somewhat smeared the envelope, widening it and reducing the gap between the peaks. While the $Q=0.58$ looks better, it would require a *MUCH* larger box to achieve or serious amounts of boost & EQ.

Example 3: Vented, PR or TL box (Butterworth 4th order high pass) and $Q=0.58$ sealed box

Frequency and step response:

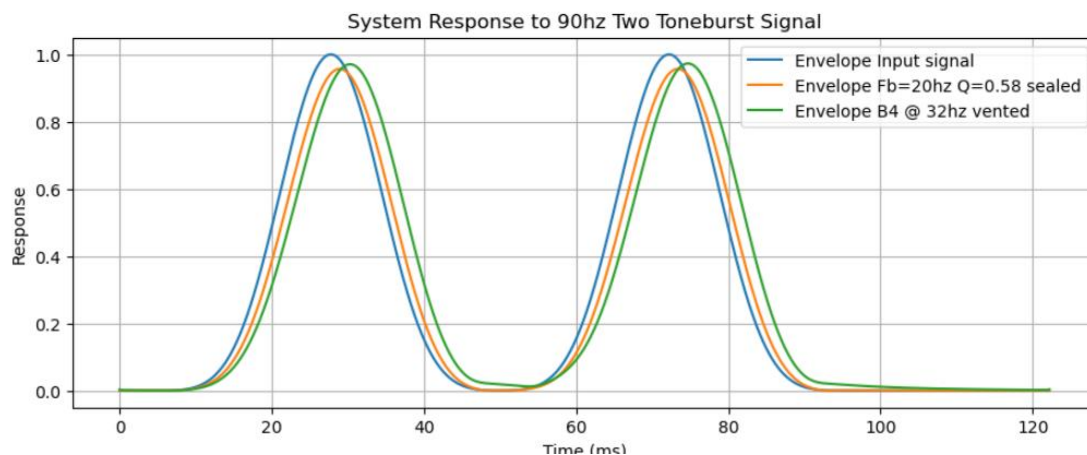


Toneburst Response Envelopes:



Notice the envelope does not fall to 0, showing the decay is inferior for the B4 alignment, this shows the superior performance of the lower frequency and lower order alignment.

I have been choosing the frequency of the toneburst so that differences are easier to see – but even at nearly triple the tuning frequency of the vented box, the sealed box has slightly better transient response:

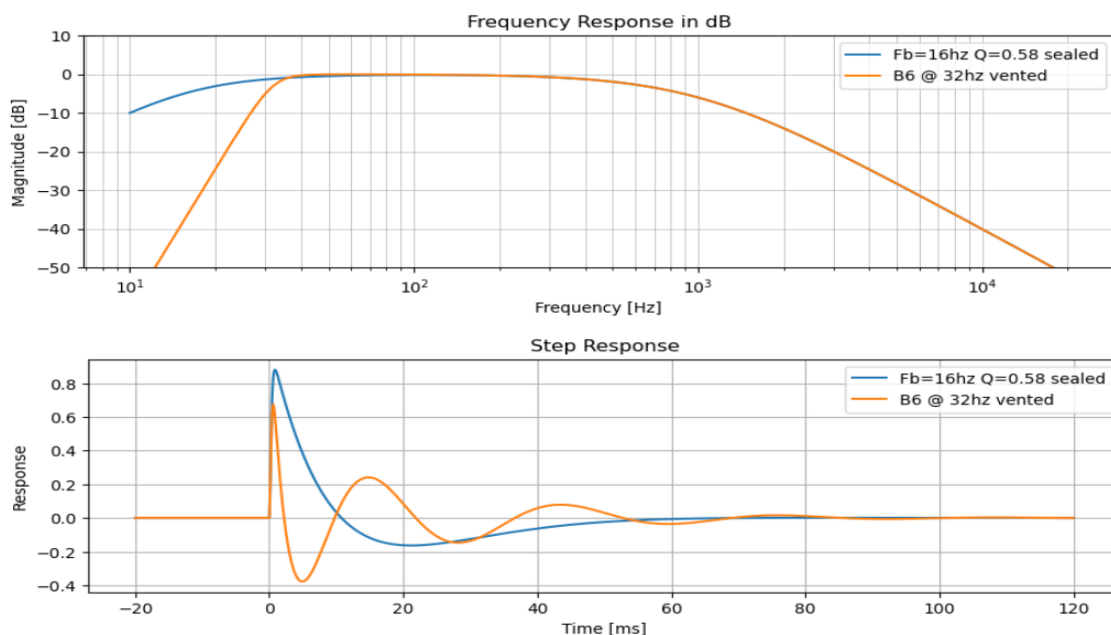


That is not a huge difference, but it's still there, very far away from the tuning frequency, note the changing horizontal time scale with the toneburst frequency change.

Final Example: Vented 6th order Butterworth vs large Q=0.58 equalized sealed box.

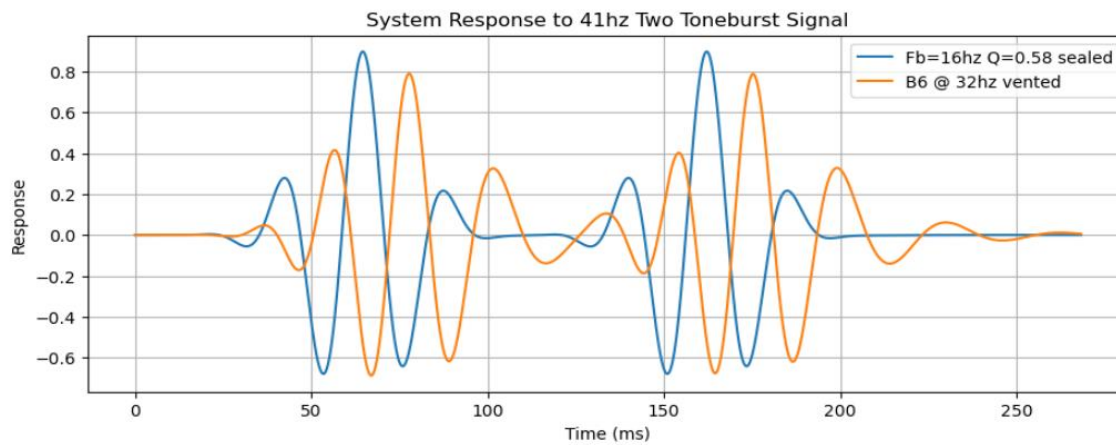
I mentioned having built a vented 6th order woofer box before and now this is a worst-case scenario: a higher-order resonant boosted bass alignment vs what I think is the optimal sealed box.

Frequency& Step response:

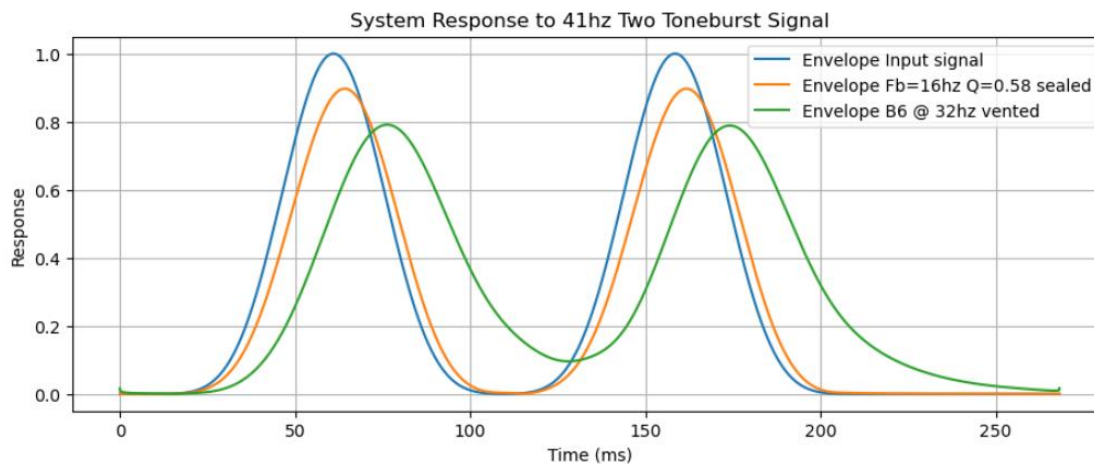


This is pretty poor performance for the B6 compared to Q=0.58.

Toneburst response:



Envelope:



The B6 system has significant delay as well as the envelope never decaying near zero between the two tonebursts. I

Final thoughts:

The higher the order of the high-pass response, the worse the transient response, within limits. For example, the $Q=1.2$ sealed box did less smearing than B4 or B6 alignments. There is never a case that extending the bass response “hurts” performance, however room gain should be measured and designed for, it’s possible a higher F_c and lower Q will be needed. When placed in a room, room modes sometimes take time to build up resonance, and having a better decay time is useful to reduce their effect.

If you are the designer who wants to use a full-range driver (to avoid having crossover cause phase rotation) in a transmission line, perhaps realizing your bass alignment is doing far more damage to transient response than a proper crossover would be in order. Speaker design is balancing tradeoffs – and these results shouldn’t be used in a religious war about sealed box vs. vented. Good sounding designs exist for each, and ultimately the audibility of the superior transient response of sealed box is still an open question.

I think for DIY having plentiful class D power and DSP it is very hard to argue against heavy EQ of a sealed box if transient performance is a goal. I would be less inclined to use in a PA setting where cooling of voice-coils and maximal output is desired, let alone the better distortion figure near box tuning (if designed correctly) a vented box can have.